Forced Convection Heat Transfer over a Horizontal Plate Embedded in a Porous Medium Saturated with a Nanofluid in the Presence of Heat Sources

Masoud Rabeti[*1], Aminreza Noghrehabadi [2], Afshin Ghanbarzadeh[3]

[*1]Department of Mechanical Engineering, Islamic Azad University, Dezful Branch, Dezful, Iran
[2, 3] Department of Mechanical Engineering, Shahid Chamran University of Ahvaz, Iran

[*1]masoud.rabeti@gmail.com; [2]noghrehabadi@scu.ac.ir; [3]ghanbarzadeh.a@scu.ac.ir

Abstract

In this paper, forced convection heat transfer of nanofluids over a horizontal flat plate embedded in a porous medium saturated with a nanofluid is numerically analyzed. In the boundary layer, heat can be generated or absorbed. It is assumed that the nanoparticles are uniformly dispersed in the base fluid. A similarity approach is used to reduce the governing partial differential equation to an ordinary differential equation. The resulting ordinary differential equation is numerically solved for a type of porous medium, sand, and three types of nanoparticles, namely, alumina (Al_2O_3), copper (Cu), and titanium dioxide (TiO_2). The effect of heat generation/absorption as well as volume fraction of nanoparticles on the heat transfer enhancement of nanofluids is theoretically investigated.

Keywords

Heat Generation/Absorption; Forced Convection; Porous Media; Nanofluid; Darcy Model

Introduction

Nowadays, nanotechnology provides a new opportunity to produce nanoparticles with dimensions below 50 nm (Wang and Mujumdar 2008). Dispersing nano-size solid particles in a conventional heat transfer fluid would increase the thermal conductivity of the fluid. Nanofluid is a fluid which contains nanometer-sized solid particles. Successful employment of nanofluids reduces the size of heat exchanger systems and also enhances the performance of these systems (Wang and Mujumdar 2008). Nanofluids have many industrial applications such as nuclear reactors, transportation, electronics as well as biomedicine and food (Kakac and Pramaumjaroenkij 2009).

Forced convection heat transfer in porous media also has many applications such as geothermal energy engineering, groundwater pollution transport, chemical reactors engineering, nuclear waste disposal, insulation of buildings and pipes (Zhao and Song 2001).

Forced convection heat transfer over a horizontal plate embedded in a porous medium has been studied in many literatures. Beckermann and Viskanta (1986) studied forced convection boundary layer flow and heat transfer along a flat plate embedded in a porous medium. Postelnicu et al. (2001) discussed the effect of variable viscosity on the forced convection flow and heat transfer past a horizontal surface placed in a saturated porous medium in the presence of internal heat generation.

Noghrehabadi et al. examined the flow and heat transfer of nanofluids over an isothermal flat plate with partial slip boundary condition (Noghrehabadi et al. 2012c), suction or injection (Noghrehabadi et al. 2012a) and magnetic field effects (Noghrehabadi et al. 2012b). Maghrebi et al. (2012) analyzed forced convection heat transfer of nanofluids in a porous channel. Rosca et al. (2012) investigated the non-darcy mixed convection of nanofluid over a horizontal surface placed in a porous medium.

Forced convection heat transfer of nanofluids over a horizontal flat plate embedded in a saturated porous medium in the presence of heat generation/absorption has not been analyzed yet. The present study aims to analyze the effect of heat generation or absorption on the forced convection heat transfer over a horizontal plate embedded in a porous medium saturated with a nanofluid. The governing partial differential equation

is transformed into an ordinary differential equation by aid of similarity variables. The governing equation, in its non-dimensional form, is a function of heat generation parameter and the ratio of thermal diffusivity of nanofluid and the base fluid.

Mathematical Formulation

Consider the steady two dimensional (x, y) forced convection heat transfer of a nanofluid over an isothermal horizontal surface which is embedded in a saturated porous medium. The schematic view of physical and coordinate system is illustrated in FIG. 1.

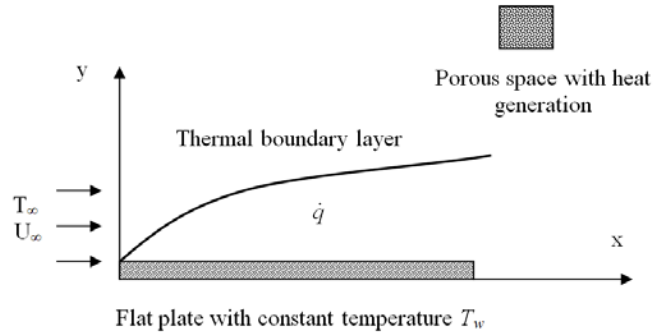

FIG. 1 PHYSICAL MODEL AND COORDINATE SYSTEM

The nanofluid flows with a constant velocity of U through the porous medium because of an external pressure gradient. The temperature of the plate is held at constant value of T_w. The nanofluid is hemogenius and the nanoparticles are uniformly dispersed in the base fluid; therefore, the concentration gradient in the boundary layer is neglected.

By assuming an isotropic Darcy porous medium and neglecting the wall effects, the governing boundary layer equations in the Cartesian coordinate system can be written as (Nield and Bejan 2006):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=-\frac{K}{\mu_{nf}}\frac{\partial P}{\partial x} \tag{2}$$

$$v=-\frac{K}{\mu_{nf}}\frac{\partial P}{\partial y} \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{m,nf}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{(\rho c)_{nf}}(T-T_\infty) \tag{4}$$

the boundary conditions are as follows:

$$y=0: \quad v=0 \qquad T=T_w \tag{5}$$

$$y\to\infty: \quad u=U \qquad T=T_\infty \tag{6}$$

where p is the pressure, u is the Darcy velocity and T is the temperature of the nanofluid. The effective thermal diffusivity of the nanofluid and porous medium is denoted by $\alpha_{m,nf}$. Here, K is the permeability of the porous medium, μ_{nf} is the dynamic viscosity of the nanofluid. Q_0 is the heat generation/absorption rate inside the boundary layer. The subscript ∞ denotes the large values outside the boundary layer. The thermo physical properties of nanofluid can be evaluated as follow:

$$\alpha_{m,nf}=\frac{k_{m,nf}}{(\rho C_p)_{nf}} \qquad \alpha_{m,f}=\frac{k_{m,f}}{(\rho C_p)_f} \tag{7-a}$$

$$(\rho C_p)_{nf}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_p \tag{7-b}$$

$$k_{eff}=k_m\left[\frac{2+k_l/k_m-2\varepsilon(1-k_l/k_m)}{2+k_l/k_m+\varepsilon(1-k_l/k_m)}\right] \tag{7-c}$$

$$\frac{k_{nf}}{k_f}=\frac{k_p+2k_f+2\varphi(k_p-k_f)}{k_p+2k_f-\varphi(k_p-k_f)} \tag{7-d}$$

Here, ε is the porosity of the porous medium, $k_{m,nf}$ is the effective thermal conductivity of nanofluid and porous medium, $k_{m,f}$ is the effective thermal conductivity of the base fluid and porous medium, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, ϕ is the nanoparticle volume fraction, ρ_f and ρ_p are the densities of the base fluid and the nanoparticles, respectively. k_m is the thermal conductivity of porous medium, k_l is the thermal conductivity of the fluid which flows through porous (nanofluid or pure fluid) and k_{nf} is the thermal conductivity of the nanofluid. k_f and k_p are thermal conductivities of the fluid and the nanoparticles, respectively. Eq. (7-c) is based on the work of Alizad et al. (2012) and Eq. (7-d) is based on the work of Bachok et al. (2011) and Khanafer and Vafai (2011). In order to evaluate $k_{m,nf,}$ first of all, k_{nf} should be evaluated from Eq. (7-d) and then the value of k_{nf} should be used as k_l in (7-c). k_{eff} is $k_{m,f}$ when k_l is the thermal conductivity of pure fluid.

Eqs. (2) and (3) are simplified using cross-differentiation, and the continuity equation is also satisfied by introducing a stream function, (ψ):

$$u=\frac{\partial\psi}{\partial y} \qquad v=-\frac{\partial\psi}{\partial x} \tag{8}$$

Solution of Eqs. (2) and (3) leads to:

$$u=U \qquad v=0 \tag{9}$$

where $\frac{\partial P}{\partial x} = -\frac{U\mu_{nf}}{K}$, and U is the free stream velocity. In order to reduce the governing equation, Eq (4), subject to the boundary conditions, Eqs. (5) and (6), to an ordinary differential equation, the similarity variables of η and θ are introduced as,

$$\eta = \frac{y}{x} Pe_x^{1/2} \tag{10}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{11}$$

local Péclet number is defined as:

$$Pe_x = \frac{Ux}{\alpha_{m,f}} \tag{12}$$

Employing the similarity variables, Eq. (10) and (11), on the Eq. (4) reduces the governing partial differential equation to the following ordinary differential equation (See Appendix A):

$$\alpha_D \theta'' + \frac{1}{2}\eta\theta' + \lambda\theta = 0 \tag{13}$$

subject to

$$\theta(0) = 1 \tag{14}$$

$$\theta(\infty) = 0 \tag{15}$$

where prime denotes differentiation with respect to η. The non-dimensional parameters of α_D and λ are defined as follows:

$$\alpha_D = \frac{\alpha_{m,nf}}{\alpha_{m,f}} \tag{16}$$

$$\lambda = \frac{Q_0 x}{(\rho c)_{nf} U} \tag{17}$$

$$\lambda_0 = \frac{Q_0 x}{U} \qquad \rightarrow \lambda = \frac{\lambda_0}{(\rho c)_{nf}} \tag{18}$$

λ_0 is the heat generation/absorption constant, and λ is the heat generation/absorption parameter, whose positive values represent heat generation and negative values represent heat absorption.

In this study, the quantity of practical interest, which shows the rate of heat transfer from the plate, is the local Nusselt number *(Nu_x)*:

$$Nu_x = \frac{x q''_w}{k_{m,f}(T_w - T_\infty)} \tag{19}$$

where

$$q''_w = -k_{m,nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{20}$$

the reduced Nusselt number is written as follow:

$$Nur = Nu_x Pe_x^{-\frac{1}{2}} = -\frac{k_{m,nf}}{k_{m,f}}\theta'(0) \tag{21}$$

Results and Discussion

In order to analyze the effect of nanoparticles on the heat transfer, three nanoparticles types, namely, Cu, Al_2O_3, and TiO_2 are chosen. The porous medium is sand, and the water is the base fluid. The required thermophysical and mechanical properties of the nanoparticles, porous medium and base fluid are shown in TABLES 1 and 2.

TABLE 1 PROPERTIES OF FLUID AND THREE NANOPARTICLES (BACHOK ET AL.2011)

Physical properties	Fluid phase (Water)	Cu	Al_2O_3	TiO_2
C_p (J/kg k)	4179	385	765	686.2
$\rho(kg/m^3)$	997.1	8933	3970	4250
k (W/m.k)	0.613	400	40	8.9538

TABLE 2 PROPERTIES OF SAND POROUS MEDIA (NIELD AND BEJAN 2006)

Physical properties	Sand
ε	0.45
k	3

The governing ordinary differential equation, Eq. (13), subject to the boundary conditions, Eqs. (14) and (15), is solved using Range-Kutta method with shooting technique. The asymptotic value of infinity is assumed as η_∞=8.

Assuming λ=0 (no heat generation/absorption) and ϕ=0 (no nanoparticle dispersed in the base fluid), the present work reduces to the work of Bejan (1984). Therefore, in order to test the accuracy of the present solution, the value of $\theta'(0)$, obtained by Bejan (1984), is compared with $\theta'(0)$ obtained in the present study in the TABLE 3. Moreover in FIG. 2, the temperature profile, obtained by Bejan (1984), is compared with $\theta(\eta)$ in the present work when λ_0=0 and ϕ=0. These comparisons show exellent agreement between the present study and the work of Bejan (1984).

The results are obtained for different values of nanoparticle concentrations as well as heat generation parameter λ.

FIG. 3 shows the effect of heat generation/absorption parameter on the temperature profiles. This figure shows that increase of η would decrease the local

temperature. The temperature on the surface is one and decreases to reach its asymptotic value of zero far from the plate. It is clear that heat generation gives an increase in the temperature of the nanofluid whereas heat absorption decreases the temperature of the nanofluid.

Variation of reduced Nusselt number with volume fraction of nanoparticles for three types of nanoparticles is shown in FIG. 4. In this figure the porous medium is sand and the λ_0=400000. FIG. 4 shows that the reduced Nusselt number would increase with an increase of nanoparticles volume fraction. It should be noted that for a constant value of nanoparticle volume fraction, the copper nanoparticles provide the highest value of reduced Nusselt number, and after that the aluminium oxide nanoparticles provide high values of reduced Nusselt number. It is worth noticing that the thermal conductivity and heat capacity of copper nanoparticles also are higher than that of titanium dioxide and aluminum oxide nanoparticles.

TABLE 3 COMPARISON BETWEEN RESULTS OF BEJAN (BEJAN 1984) AND PRESENT WORK FOR (λ_0=0 AND Φ=0)

$\theta'(0)$	
Bejan(bejan 1984)	Present work
0.564	0.564

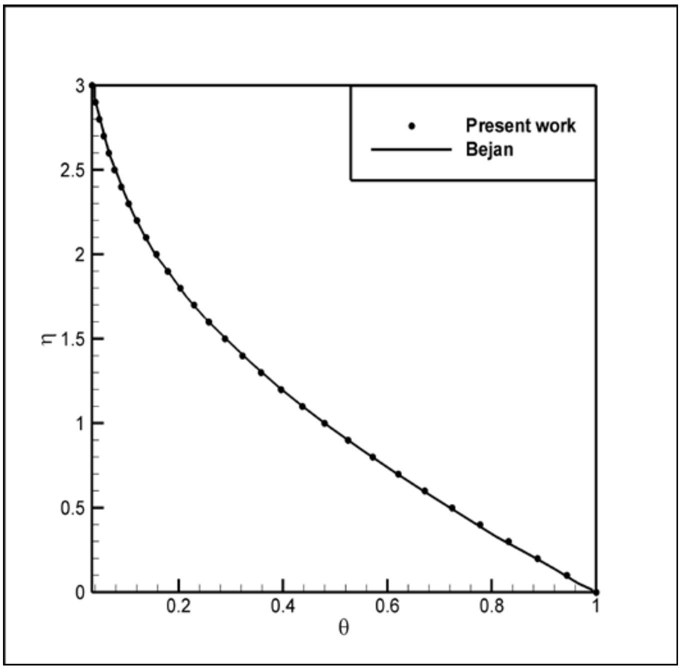

FIG. 2 COMPARISON BETWEEN TEMPERATURE PROFILE OF PRESENT STUDY AND THAT OF BEJAN (1984)

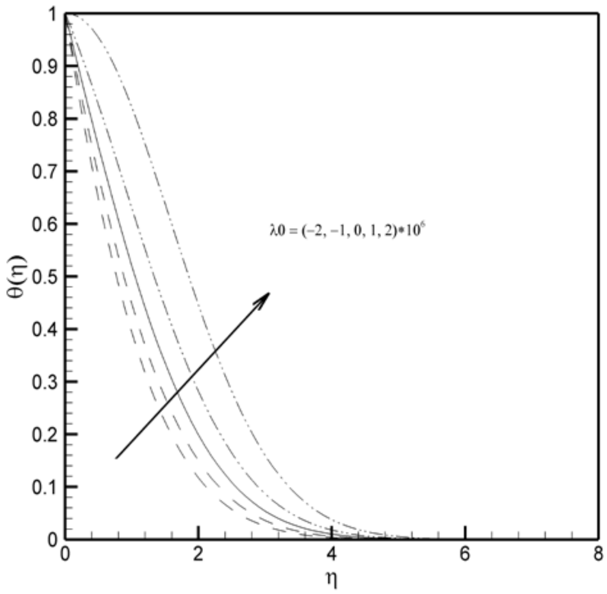

FIG. 3 EFFECTS OF HEAT GENERATION PARAMETER ON TEMPERATURE PROFILES FOR SAND POROUS MEDIA AND AL_2O_3 NANOPARTICLES WHEN Φ=0.2

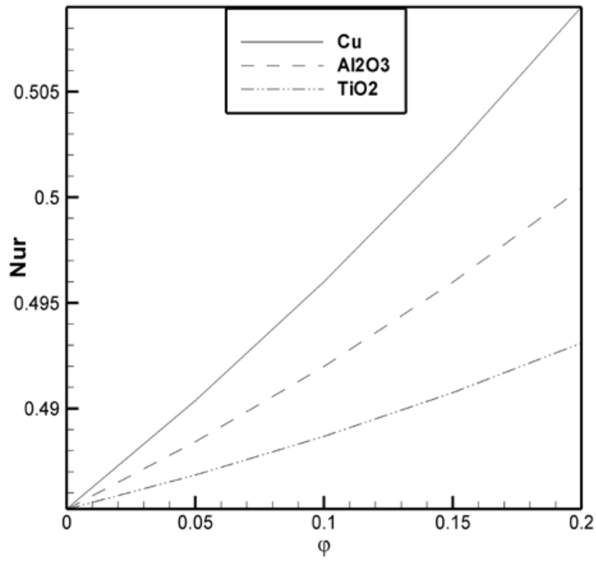

FIG. 4 VARIATION OF REDUCED NUSSELT NUMBER WITH Φ FOR SAND AND THREE TYPES OF NANOPARTICLES WHEN λ_0=400000

Conclusion

Effect of heat generation or absorption as well as types and concentration of nanofluids on the forced convection heat transfer over a horizontal flat plate embedded in a porous medium is investigated numerically. The results can be summarized as follows:

1. Heat generation increases temperature of nanofluid whereas heat absorption decreases the temperature.

2. For all studied cases, an increase of nanoparticles volume fraction would significantly increase the reduced Nusselt number.

REFERENCES

Alizad, K. Vafai, K. and Shafahi, M. ""Thermal performance and operational attributes of the startup characteristics of flat-shaped heat pipes using nanofluids." International Journal of Heat and Mass Transfer, 140–155, 2012.

Bachok, N. Ishak, A. and Pop, I. ""Flow and heat transfer characteristics on a moving plate in a nanofluid." International Journal of Heat and Mass Transfer, 2011.

Beckermann, C. and Viskanta, R. ""Forced convection boundary layer flow and heat transfer along a flat plate embedded in a porous medium." ht. J. Hear Mass Transfer, 1547-1551, 1986.

Bejan, A. "Convection Heat Transfer." Wiley, 1984.

Kakac, S. and Pramaumjaroenkij, A. ""Review of convective heat transfer enhancement with nanofluids." Int. J. Heat Mass Transfer, 3187–3196, 2009.

Khanafer, K. and Vafai, K. ""A critical synthesis of thermophysical characteristics of nanofluids." International Journal of Heat and Mass Transfer, 4410–4428, 2011.

Maghrebi, M. J. Nazari, M. and Armaghani, T. ""Forced convection heat transfer of nanofluids in a Porous channel." Transp Porous Med, 401–413, 2012.

Nield, D. A. and Bejan, A. "Convection in Porous Media." Springer, 1_640, 2006.

Noghrehabadi, A. Ghalambaz, M. Ghalambaz, M. and Ghanbarzadeh, A. ""Comparing thermal enhancement of Ag-water and SiO_2-water nanofluids over an isothermal stretching sheet with suction or injection." Journal of Computational and Applied Research in Mechanical Engineering, 35-47, 2012a.

Noghrehabadi, A. Ghalambaz, M. and Ghanbarzadeh, A. "" Heat transfer of magnetohydrodynamic viscous nanofluids over an isothermal stretching sheet." Journal of Thermophysics and Heat Transfer, 686-689, 2012b.

Noghrehabadi, A. R. Pourrajab, R. and Ghalambaz, M. ""Effect of partial slip boundary condition on the flow and heat transfer of nanofluids past stretching sheet prescribed constant wall temperature." International Journal of Thermal Sciences, 253-261, 2012c.

Postelnicu, A. Grogan, T. and Pop, I. ""The effect of variable vscosity on forced convection flow ast a horizontal flat plate in a porous medium with internal heat generation." Mechanics Research Commumcations 331_337, 2001.

Rosca, A. V. Rosca, N. C. Grosan, T. and Pop, I. ""Non-Darcy mixed convection from a horizontal plate embedded in a nanofluid saturated porous media." International Communications in Heat and Mass Transfer, 1080–1085, 2012.

Wang, X. Q. and Mujumdar, A. S. ""A review on nanofluids - part i: theoretical and numerical investigations. " Brazilian Journal of Chemical Engineering, 613-630, 2008.

Zhao, T. S. and Song, Y. J. ""Forced convection in a porous medium heated by a permeable wall perpendicular to flow direction: analyses and measurements." International Journal of Heat and Mass Transfer, 1031–1037, 2001.

Nomenclature	
specific heat at constant pressure	C_p
permeability	K
thermal conductivity	k
local Nusselt number	Nu_x
local Péclet number	Pe_x
fluid temperature	T
ambient temperature	T_∞
velocity components	u,v
Cartesian coordinates	x,y
Greek symbols	
thermal diffusivity	α
porosity	ε
similarity variable	η
dimensionless temperature	θ
heat generation/absorption parameter	λ
dynamic viscosity	μ
density	ρ
nanoparticle volume fraction	φ
Subscripts	
ambient condition	∞
fluid	f
porous media	m
effective value for porous medium and base fluid	m,f
effective value for nanofluid and porous medium	m,nf
nanofluid	nf
particle	p
Superscript	
differentiation with respect to η	$'$

Appendix A:

Details of transforming the PDE (Energy equation) to ODE:

Differentiation η with respect to x and y:

$$\frac{\partial\eta}{\partial x}=-\frac{1}{2}y(\frac{U}{\alpha_{m,f}})^{1/2}x^{-3/2}\qquad \frac{\partial\eta}{\partial y}=\frac{Pe_x^{1/2}}{x} \tag{A1}$$

and differentiation T with respect to x and y:

$$\frac{\partial T}{\partial x}=(T_w-T_\infty)\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}=-\frac{1}{2}(T_w-T_\infty)y(\frac{U}{\alpha_{m,f}})^{1/2}x^{-3/2}\theta' \tag{A2}$$

$$\frac{\partial T}{\partial y}=(T_w-T_\infty)\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial y}=(T_w-T_\infty)\frac{Pe_x^{1/2}}{x}\theta' \tag{A3}$$

$$\frac{\partial^2 T}{\partial y^2}=(T_w-T_\infty)(\frac{U}{\alpha_{m,f}})^{1/2}x^{-1/2}\frac{\partial\theta'}{\partial\eta}\frac{\partial\eta}{\partial y}=$$

$$(T_w-T_\infty)(\frac{U}{\alpha_{m,f}})^{1/2}x^{-1/2}(\frac{U}{\alpha_{m,f}})^{1/2}x^{-1/2}\theta''=$$

$$(T_w-T_\infty)\frac{U}{\alpha_{m,f}x}\theta'' \tag{A4}$$

Energy equation follows as:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{m,nf}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{(\rho c)_{nf}}(T-T_\infty) \tag{A5}$$

Substituting differentiations into energy equation:

$$U(-\frac{1}{2})(T_w-T_\infty)y(\frac{U}{\alpha_{m,f}})^{\frac{1}{2}}x^{-\frac{3}{2}}\theta'=\alpha_{m,nf}(T_w-T_\infty)\frac{U}{\alpha_{m,f}x}\theta''$$

$$+\frac{Q_0}{(\rho c)_{nf}}(T_w-T_\infty)\theta\rightarrow-\frac{1}{2}\eta\theta'=\frac{\alpha_{m,nf}}{\alpha_{m,f}}\theta''+\frac{Q_0 x}{(\rho c)_{nf}U}\theta$$

$$\rightarrow\alpha_D\theta''+\frac{1}{2}\eta\theta'+\lambda\theta=0 \tag{A6}$$